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Scaling of vortex transport properties and nonlinear ac response of high-temperature superconductors

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Abstract. The macroscopic behavior of high-temperature superconductors is described by a nonlinear response function in combinations with Maxwell equations. This function is compatible with the suggested different model pinning barriers $U(J)$. A comparison of this function to the scaling behavior of the isothermal current-voltage characteristics measured in twinned $YBa_2Cu_3O_{7-\delta}$(YBCO) samples shows fair agreement. We also compare the amplitude dependence of ac susceptibility derived from this function with several experimental results of high-temperature superconductors and find a general power law in the out-of-phase χ'' peak shift.

PACS. 74.25.Fy Transport properties (electric and thermal conductivity, thermoelectric effects, etc.) – 74.25.Nf Response to electromagnetic fields (nuclear magnetic resonance, surface impedance, etc.)

1 Introduction

The importance of thermal fluctuations, the characterized short coherence length and the large anisotropy of high-temperature superconductors (HTSC) give rise to a vortex system with a number of interesting features. The static and dynamic response of such a system has been the subject of numerous recent experimental and theoretical investigations [1]. The analysis of current-voltage *I-V* characteristics measured in twinned $YBa_2Cu_3O_{7-\delta}$(YBCO) [2,3] with the magnetic field applied in the c direction, in terms of the vortex-glass scaling theory [4], provide impressive evidence of a second order phase transition. However, Nelson and Vinokur questioned whether these transitions are really caused by uncorrelated disorder as assumed in the original vortex-glass phenomenology [4] because twin boundaries offer much stronger pinning [5]. The low temperature glassy phase stabilized by correlated defects like twin boundary, columnar pinning center etc. is called as Bose-glass. For fields parallel to c axis, the Bose-glass theory predicts for the *I-V* characteristics with similar critical exponent relation as given by the vortex-glass theory. Another attempt to explain the experimental data of Koch *et al.* in reference [2] has been made by Coppersmith *et al.* [6] in terms of the flux-creep-flow model [7]. Though this model reproduces the qualitative features of the data, it fails to give a quantitative fit. The exponent values needed to collapsed the *I-V* curves in the model of Coppersmith *et al.* are $z = 13.5$ and $\nu = 0.6$ [7] in contrast to $z = 4.8$ and $\nu = 1.7$ from the experimental data of reference [2]. Thus, this reinterpretation has been reasonably questioned [8]. Parallel to the dc transport, the ac magnetic susceptibility measurement is also a powerful method of investigating the behavior of vortex system. The application of a time-dependent field $H(t) = H_0 + h_{ac}e^{-i\omega t}$ to the sample surface results in an electric-field gradient in the sample interior (h_{ac} is the ac-field amplitude and ω the angular frequency). This gives rise to a shielding current, which in turn exerts a Lorentz force on the vortices in the sample. The measurements of such ac response contain much valuable information about pinning and creep of vortices, and turn out to be useful to test models describing the ac losses which have to be carefully characterized and monitored for many applications. The controversy over the analysis of the ac response is often noteworthy because of the complicate interplay of hysteretic and eddy-current losses [9].

Generally speaking, the equations that describe the behavior of the vortex system on a macroscopic scale are the Maxwell equations combined with the materials equation of superconductors $J(E, B, T)$. Various models suggested in literature correspond to different specific forms of the materials equation.

In the case of ideal type-II superconductors with negligible flux pinning, the material can be characterized by the equation $E = \rho_f(B, T)J$ with $\rho_f \approx \rho_n B/B_{c2}$, the flux-flow resistivity as estimated by Bardeen and Stephen [10]. On the other hand, in nonideal superconductors with considerable pinning, the material is described by a set of equations $\mathbf{E} = \mathbf{B} \times \mathbf{v}$, $\mathbf{v} = \mathbf{v}_0 \exp(-U(J)/kT)$ or $E(J) = J\rho_f e^{-U(J)/kT}$, where the activation barrier U additionally depends on the temperature T and magnetic field B. Different types of $U(J)$ have been

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suggested to approximate the real barrier, for instance, the Anderson-Kim model [7] with $U(J) = U_c(1 - J/J_c)$, the logarithmic barrier $U(J) = U_c \ln(J_c/J)$ [11] and the inverse power-law with $U(J) = U_c[(J_c/J)^\mu - 1]$ [4,12,13].

In present work, we want to show that both dc transport and ac response of HTSC can be described consistently by using a unified materials equation [14,15]. In subsequent section, we briefly introduce the nonlinear response function and show its connection with the critical-state model. In Section 3, we compare this equation with the widely quoted transport experimental results of Koch *et al.* [2,8]. A further discussion of the critical current and ac response is given in Section 4. Finally we conclude by a short summary.

2 Nonlinear response function

In view of the finding of Bardeen and Stephen [10], for the steady-state of flux motion in nonideal type-II superconductor the mean transport current density J can be phenomenologically expressed as

$$J = J_p + J_f \tag{1}$$

with

$$J_f \equiv E(J)/\rho_f \tag{2}$$

the component due to the moving vortices of uniform density. J_p is the contribution from the pinned vortices.

We find, if one makes a common modification to the different model barriers $U(J)$ as

$$U(J) \to U(J_p \equiv J - E/\rho_f), \tag{3}$$

the corresponding modified materials equation

$$E(J) = J\rho_f e^{-U(J_p)/kT} \tag{4}$$

leads to a common normalized form as

$$y = x \exp[-\gamma(1 + y - x)^p] \tag{5}$$

with x and y the normalized current density and electric field respectively. γ is a parameter characterizing the symmetry breaking of the pinned vortices system and p is an exponent.

To show the connection of the nonlinear response function equation (5) with the critical-state model $U(J)$, we start from the expression widely used for flux creep with the logarithmic barrier [11],

$$E(J) = \rho_f J \exp\left[-\frac{U_c}{kT} \ln\left(\frac{J_{c0}}{J}\right)\right] \cdot \tag{6}$$

Substituting $J_p \equiv J - E(J)/\rho_f$ for the current density J in the bracket on the right-hand side of equation (6), we get

$$E(J) = \rho_f J \exp\left[-\frac{U_c}{kT} \ln\left(\frac{J_{c0}}{J_p}\right)\right] \cdot \tag{7}$$

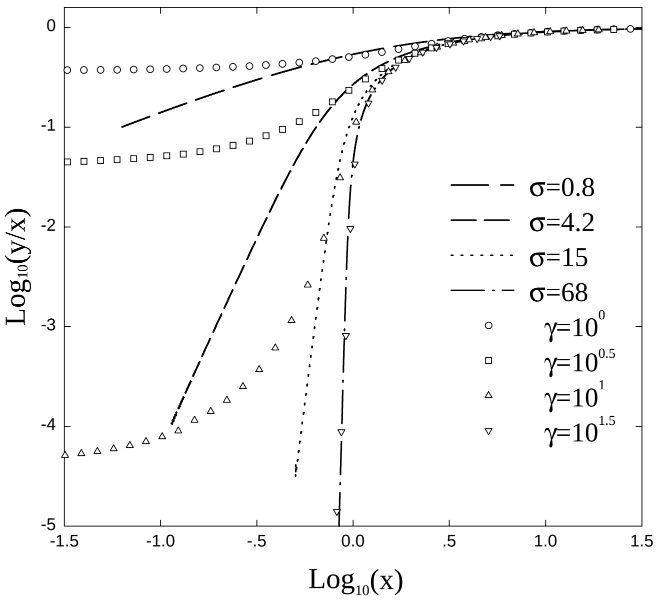


Fig. 1. Numerical solutions of equation (5) (open symbols) and equation (11) (lines) for comparison.

The definition of barrier implies $J_{c0} \geq J_p$. Using the approximation

$$\ln \eta = \sum_{n=1}^{\infty} \frac{1}{n}(1 - \eta^{-1})^n \approx a(1 - \eta^{-1})^p, \quad \left(\eta > \frac{1}{2}\right), \tag{8}$$

finally we find equation (7) in the form

$$\ln\left(\frac{x}{y}\right) = \gamma(1 + y - x)^p, \tag{9}$$

which is the general normalized form of the materials equation (5). Here we have

$$\gamma \equiv a\frac{U_c}{kT} \quad x \equiv \frac{J}{J_{c0}} \quad y \equiv \frac{E(J)}{\rho_f J_{c0}} \cdot \tag{10}$$

In earlier works, this materials equation for type-II superconductors has also been shown in connection with the Anderson-Kim model and the inverse power-law $U(J)$ [14,15].

The numerical factor a in the approximation equation (8) should be evaluated with considering the limitation of sample size to the realistic barrier $U(J)$ as discussed in references [13–15]. Considering this limitation as a cut-off of the series in equation (8), we have

$$a = \sum_{n=1}^{N_c} \frac{1}{n} = C + \ln N_c,$$

where C is the Euler constant and N_c corresponds to the realistic cut-off of the series in equation (8). Usually a is of the order 2-4. Ignoring this limitation, one gets from equation (7) an even simpler expression

$$E(J) = \rho_{\rm f} J \left(\frac{J_{\rm p}}{J_{\rm c0}}\right)^{U_{\rm c}/kT}$$

or

$$y/x = (x-y)^{\sigma} \tag{11}$$

with $\sigma = U_{\rm c}/kT$, though the latter can not be used to interpret the case with small barrier and thermally activated flux-flow (TAFF). In Figure 1, we show the numerical solutions of equations (5, 11) for comparison.

Equation (5) gives a relation between the parameter γ and the slope

$$S \equiv \frac{{\rm d}\ln y}{{\rm d}\ln x} = \frac{1 + p\,\gamma\,x(1+y-x)^{p-1}}{1 + p\,\gamma\,y(1+y-x)^{p-1}}\,\cdot \tag{12}$$

The maximal slope $S_{\max}$ occurs at the inflection point $(x_i,\, y_i)$ of isotherm $\ln y - \ln x$, where

$$p^2\gamma^2 x_i y_i (1+y_i-x_i)^{2p-1} = 1 - p(x_i - y_i), \tag{13}$$

and we have the power law $V \propto I^{S_{\max}}$.

From equations (12, 13) we get

$$S_{\max} = \frac{1+\gamma x_i\zeta}{1+\gamma y_i\zeta} \approx \left(\frac{x_i}{y_i}\right)^{1/2} \tag{14}$$

and

$$\gamma^2 x_i y_i = 1/\zeta^2, \tag{15}$$

where $\zeta^2 \equiv p^2(1+y_i-x_i)^{2p-1}/[1-p(x_i-y_i)] \approx 1$. In view of the numerical solution of equation (5) as shown in Figure 1, one finds $x_i \approx 1$. Thus from equations (14, 15), we have approximately the relation

$$\gamma \approx 2S_{\max}^2 \ln S_{\max}. \tag{16}$$

3 Scaling behavior of isothermal E(J) curves

Now we compare our equation (5) with the scaling behavior of the experimentally measured isothermal $E(J)$ curves obtained by Koch *et al.* with YBCO samples [2,8]. At different temperatures and magnetic fields, they found that for each field at a single well defined temperature T, the I-V curve shows a power-law behavior $V \propto I^S$. This temperature is defined as $T_{\rm g}$. All the isotherms can be collapsed onto two scaling functions, for $T > T_{\rm g}$ and $T < T_{\rm g}$ correspondingly, by plotting V/I scaled by $|T-T_{\rm g}|^{\nu(z-1)}$ *vs.* I scaled by $|T-T_{\rm g}|^{2\nu}$, where ν is the exponent of the coherence length ξ, $\xi \sim |T-T_{\rm g}|^{-\nu}$, and z is the dynamical exponent of the coherent time ξ^z. Based on their experimental data they found $\nu = 1.7$ and $z = 4.8$ for $B = 2$, 3 and 4 T. On the basis of correlated pinning theory, Nelson and Vinokur predicted another scaling relation between the current density J and electric field E. When magnetic field is aligned with the correlated defects, which is the case of the upper mentioned experiments [2,8], this relation has the form

$$E|t|^{-\nu'(z'+1)} = F_{\pm}(|t|^{-3\nu'} J\phi_0/C), \tag{17}$$

where ν' and z' are the exponents governing the size and time relaxation of fluctuations respectively, $t = (T - T_{\rm BG})/T_{\rm BG}$, $T_{\rm BG}$ is the Bose-glass transition temperature and $F_{\pm}$ represents two analytic functions for $t > 0$ and $t < 0$ respectively [5]. Nelson and Vinokur found that the data in references [2,8] fit the Bose-glass scaling relation (17) with $\nu' \approx 1.3 \pm 0.5$ and $z' \approx 7 \pm 2$.

In equation (10) of our previous section, the energy $U_{\rm c}$ is temperature and field dependent, and is proportional to the vortex kink energy $E_{\rm k}$ of the Bose-glass theory, $U_{\rm c} \propto E_{\rm k} \equiv d\sqrt{\tilde{\varepsilon}_1 U_0}$ (see Ref. [5]). Therefore, one may reasonably assume $E_{\rm k}(T) \propto (T^*-T)^{\delta}$, $J_{\rm c0}(T) \propto (T^*-T)^{\alpha}$, and $\rho_{\rm f} \propto T$ with T^* being the irreversibility temperature where tilt modules $\tilde{\varepsilon}_1$ vanishes, so according to equation (10) we expect

$$\gamma(T) = \gamma_0 (T^*-T)^{\delta-\alpha p}/kT, \tag{18}$$

$$I \propto xJ_{\rm c0}(T) \propto x(T^*-T)^{\alpha}, \tag{19}$$

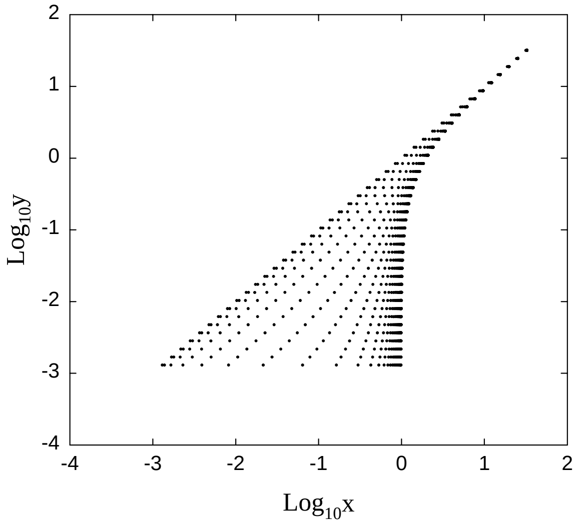
$$V \propto yJ_{\rm c0}(T)\rho_{\rm f} \propto y(T^*-T)^{\alpha}T. \tag{20}$$

In accordance with the observed $S_{\max} \approx 2.5$ at 4 T [2], one may expect $\gamma(T_{\rm BG}) \approx 11.5$. Assuming $\delta = 2.5$, $\alpha = 3$, and $p = 0.6$, we get from equation (5) more than 100 $E(J)$ isotherms near the $T_{\rm BG} \approx 78$ K (as observed in Refs. [2,8]). These curves are shown in Figure 2a. All the isotherms collapsed nicely onto two curves ($T > T_{\rm BG}$ and $T < T_{\rm BG}$), consistent with the scaling of $\nu' \sim 1.2$, and $z' \sim 7.2$, as shown in Figure 2b, in fair agreement with the similar Bose-glass scaling result of the data in references [2,8].

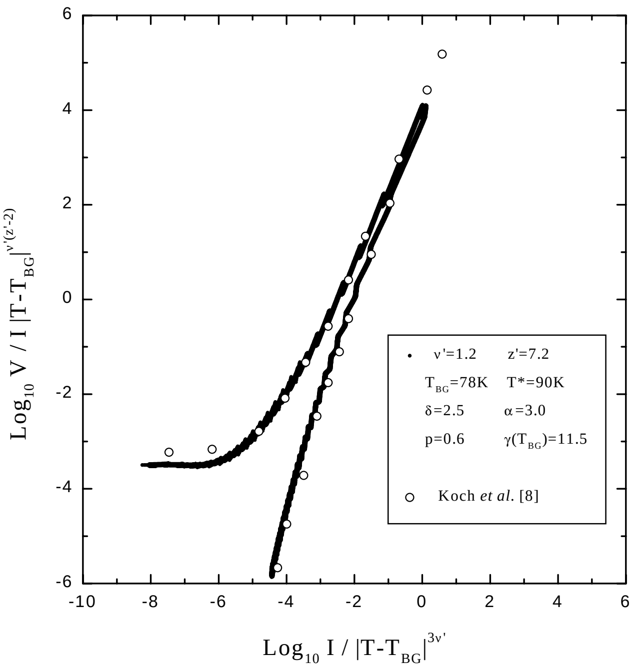
4 Critical current and ac response

Most experiments with HTSC deal with thin flat sample in a perpendicular magnetic field. Critical state models like Bean model and its modifications are often used to analyze the results where the hysteretic losses dominate. However some problems with the amplitude dependence of ac response still remain unsolved [9,17]. Several common features of the amplitude dependence of χ have been experimentally observed for different kinds of materials: a) a parallel shift of the in-phase susceptibility $\chi' - T$ curve with increasing $h_{\rm ac}$ toward lower temperatures is observed; b) the onset of diamagnetism and dissipation does not appear to depend on $h_{\rm ac}$ values; c) the out-of-phase χ'' peak shifts to lower temperatures with increasing $h_{\rm ac}$ and broadens in the low temperature side; d) the absorption peak increases slightly when $h_{\rm ac}$ increases [9,16–20].

Since different loss mechanisms are not simply additive [18], it is desirable to have a unified consistent description for the critical current. On the basis of our nonlinear



(a)



(b)

Fig. 2. (a) The *I*-*V* curves derived from equation (5). (b) (•) Scaling of the isotherms in (a) with the Bose-glass scaling relationship of Nelson and Vinokur in reference [5]. (○) The original experimental result of reference [2].

response function (5, 10), it is easy to express explicitly the critical current density J_c by a certain criterion of electric field E_c as $E(J_c) \equiv E_c$ in the form

$$J_c = J_{c0}x_c = J_{c0}\left[1 - \gamma^{-1/p}\left[\ln\left(\frac{x_c}{y_c}\right)\right]^{1/p} + y_c\right] \quad (21)$$

with $y_c = E_c/J_{c0}\rho_f$. In the magnetization procedure, E_c corresponds to the electric field $\mathbf{E}(\mathbf{r}, t)$ due to the certain sweep rate of the magnetic field.

Unlike the ideal Bean model, the critical current density J_c defined by our materials equation as equation (21)

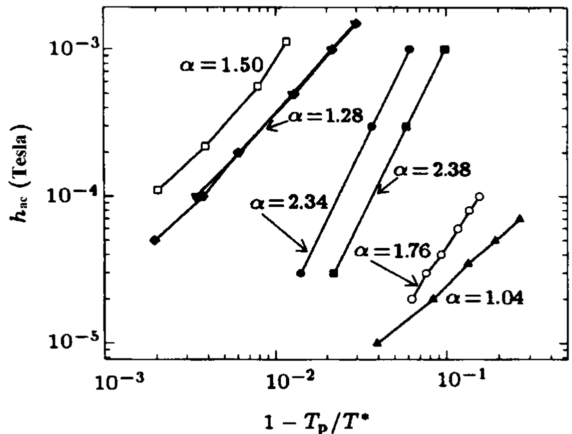


Fig. 3. Plots of h_{ac} as a function of $(1 - T_p/T^*)$ in the above experiments. T_p is the temperature at the peak of χ''. The (□) for YBCO bulk sample [19], (•) for single crystal of $Pr_{1.85}Ce_{0.15}CuO_{4-y}$ at $f = 111$ Hz and $\mu_0 H = 1$ T [20], (■) for single crystal of $Pr_{1.85}Ce_{0.15}CuO_{4-y}$ at $f = 111$ Hz and $\mu_0 H = 0.1$ T [20], (▼) for a disk (diameter 1 mm) of YBCO film [17], (♦) for a rectangle (2×3 mm^2) of YBCO film [17], (○) for a ring (width 50 μm) of YBCO film [17], (▲) for a ring (width 25 μm) of YBCO film [17].

increases slightly with increasing E_c which is proportional to the amplitude h_{ac} for a given frequency ω. Since the saturation magnetic moment is proportional to J_c for various samples and configurations as discussed in reference [17], the observed amplitude dependence of χ''_{max} can be reasonably understood.

The peak in χ'' occurs just when the ac current has penetrated the sample to a distance $d/2$. Thus the criterion for the peak in susceptibility is

$$\frac{ch_{ac}}{2\pi d} = J_c(T_p) = J_{c0}(T_p)\left[1 - \gamma^{-1/p}(T_p)\left[\ln\left(\frac{J_c\rho_f}{E_c}\right)\right]^{1/p} + \frac{E_c}{J_{c0}\rho_f}\right]. \quad (22)$$

This relation characterizes the peak position shift effected by the amplitude h_{ac}.

We find that the experimentally measured χ'' peak position T_p can be approximately described by a general phenomenological relationship as

$$h_{ac} \propto [T^* - T_p(h_{ac})]^{\alpha}, \quad (23)$$

where $T^* \equiv T_p(h_{ac} = 0)$, and α is a numerical exponent. This power law shift of T_p to lower values by increasing amplitude h_{ac} is consistent with our equation (22), which shows

$$h_{ac} \propto J_{c0}(T_p), \quad (24)$$

and we have the relation $J_{c0}(T) \propto (T^* - T)^{\alpha}$ in equation (19) of Section 3.

In Figure 3, the experimental data of χ''_{max} peak position T_p from different references in literature are summarized, where we see the power law equation (23) holds in general.

The above discussion can be applied straightforwardly to estimate the ac losses, since, for a given amplitude of the ac magnetic field, the loss per cycle is proportional to the imaginary part of the complex permeability μ'' [18].

5 Summary

We show a nonlinear response function for describing the macroscopic electromagnetic properties of high temperature superconductors with inhomogeneities or defects as pinning centers. This function is compatible with different suggested model barriers $U(J)$ and able to make a consistent description of the vortex system near transition. It is useful for understanding the experimental results of both transport properties and ac susceptibility measurements.

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